

Final report for the project
"SHAPE SENSITIVITY ANALYSIS OF STATIC AND DYNAMIC
AEROELASTIC RESPONSES"

8011

Contract Number: NAG-1-1411
Grant Monitor: Jean-Francois M. Barthelemy
Principal Investigator: Rakesh K. Kapania

This is a brief description of the research work done in this project. For details, please refer to the publications attached.

Sensitivity Analysis of the Static Aeroelastic Response of a Wing

A technique to obtain the sensitivity of the static aeroelastic response of a three dimensional wing model to the variations in various shape parameters namely: (i) wing area, (ii) sweep, (iii) aspect ratio, and (iv) taper ratio, is designed and implemented. The aeroelastic responses were the generalized aeroelastic displacements and the trim angle of attack. The formulation is quite general so that it is applicable with any aerodynamic code which, for a given geometry and structural deformations, provides aerodynamic pressures on the wing surface. A program to combine the discipline level, or local, sensitivities into global sensitivity derivatives is developed. The sensitivities were obtained by differentiating the constitutive equations. It was shown that the resulting sensitivity equations can be reformulated into a variation of the Sobieski's Global Sensitivity Equations (G.S.E) approach. Both schemes gave the various global sensitivities (i.e., the sensitivity including all interdisciplinary interactions) in terms of local sensitivities (i.e., the sensitivities obtained at the discipline level).

A scheme is developed to interface arbitrary aerodynamic and structural codes in order to calculate the aeroelastic response of a wing. Anticipating the availability of nonlinear aerodynamic models in the future, the formulation does not assume a linear dependence between the lift generated and the generalized coordinates and the initial angle of attack. This formulation needs an iterative process to calculate the angle of attack to which the aircraft is trimmed to produce the required lift. The aeroelastic problem is broken into subproblems (or blocks) by discipline. The aerodynamic and structural blocks are called iteratively to produce a converged static wing loading and shape. Shape sensitivity values for this converged wing are then obtained.

The aerodynamic block is responsible for generating the loads on the wing. It reads as input the wing geometry parameters and the current wing deflections. It is able to output the pressure on the wing at arbitrary points. The aerodynamic analysis in this study is performed by program FAST, which is a lifting-surface code based on kernel-function theory. The structural block is responsible for calculating the deflection of the wing. It is given the wing geometry and wing loading. It calculates the deflected shape of the wing. Giles' equivalent plate code ELAPS is used in this study to perform the structural analysis and is based on the Ritz method.

A variety of representations of the wing pressure field are developed and tested to determine the most accurate and efficient scheme for representing the field outside of the aerodynamic code. Chebyshev polynomials are used to globally fit the pressure field. This approach had some difficulties in representing local variations in the field. It was found that the generalized pressure coefficients, due to the global nature of the interpolation polynomials, may be sensitive to small changes in independent variables. As

a result, the determination of the local derivatives of some of the generalized aerodynamic coefficients was found to be difficult when forward or central differences were used. A higher order finite difference scheme using a large step size was employed so that the effect of local wiggles can be reduced, which is an expensive option. So a variety of local interpolation polynomial pressure representations are also implemented. These panel based representations use a constant pressure value, a bilinearly interpolated value, or a biquadratically interpolated value. The interpolation polynomial approaches do an excellent job of reducing the numerical problems of the global approach for comparable computational effort. Regardless of the pressure representation used, sensitivity and response results with excellent accuracy have been produced for large integrated quantities such as wing tip deflection and trim angle of attack. The sensitivities of such things as individual generalized displacements have been found with fair accuracy. In general, accuracy is found to be proportional to the relative size of the derivatives to the quantity itself.

Shape Sensitivity Analysis of Divergence Dynamic Pressure

In the design phase of an engineering system, it is very important to know how perturbing a certain input parameter will affect an output variable of concern. These sensitivities can help the designer optimize a configuration with respect to certain input parameters. In an aircraft design problem, the availability of the sensitivity derivatives of a quantity which governs divergence (a static aeroelastic instability), such as the divergence dynamic pressure is beneficial to the design. This study has been conducted to determine the sensitivity of divergence dynamic pressure of a wing with respect to (i) sweep, (ii) wing area, (iii) aspect ratio, and (iv) taper ratio. The formulation is quite general so that it may be used in conjunction with any aerodynamic and structural analysis packages.

The Giles' equivalent plate model (ELAPS) has been used for modeling the wing structure and the subsonic kernel function aerodynamics (FAST) has been used to generate the generalized pressure coefficients. The divergence dynamic pressure is found from the eigensolution of a system of equations which comprise the stiffness matrix of the wing structure and aerodynamic kernel matrix. The sensitivities in this study are calculated semi-analytically. The sensitivity prediction of the divergence dynamic pressure with respect to shape parameters are found to be good, giving a linear approximation to the divergence dynamic pressure variation with changes in the parameter that is perturbed.

Sensitivity Analysis of Wing Aeroelastic Responses

Design for prevention of aeroelastic instability (that is, the critical speeds leading to aeroelastic instability lie outside the operating range) is an integral part of the wing design process. Availability of the sensitivity derivatives of the various critical speeds with respect to shape parameters of the wing could be very useful to a designer in the initial design phase, when several design changes are made and the shape of the final configuration is not yet frozen. These derivatives are also indispensable for a gradient-based optimization with aeroelastic constraints.

This study has examined the flutter characteristic of a typical section in subsonic compressible flow. Indicial response functions are used for the normal force and pitching moment coefficients of the two degree of freedom airfoil and the unsteady aerodynamics is represented by a state-space model. The aeroelastic equations are solved as an eigenvalue problem to determine the onset of aeroelastic instability. The sensitivity of the

flutter speed of the typical section with respect to its mass and stiffness parameters, namely, mass ratio, static unbalance, radius of gyration, bending frequency and torsional frequency are calculated analytically. The sensitivity derivatives show good agreement with finite difference derivatives. The aeroelastic equations are also integrated with respect to time at different values of freestream speed using the Wilson- θ method, to observe the aeroelastic phenomena in real time. The stable, neutrally stable and unstable nature of the aeroelastic response can be seen at speeds below, at and above the flutter speeds, respectively, for the typical section.

The above aerodynamics for a typical section was extended to develop a strip-theory formulation representing the unsteady aerodynamic forces on a wing. The structural modeling is done using classical plate theory and is based on a Rayleigh-Ritz formulation using Chebyshev polynomials for the wing displacements. The structural equations are coupled with the time-domain aerodynamic equations to formulate the aeroelastic equations as a set of first-order ordinary differential equations. These equations are solved as an eigenvalue problem to determine the critical speed of the wing. The natural frequencies and flutter speeds are compared with previously published experimental values obtained from wind tunnel tests and the results agree fairly well. The sensitivity of divergence and flutter speeds to shape parameters, namely, aspect ratio, area, taper ratio and sweep angle are computed analytically. These derivatives have been accurately evaluated and they form a tangent to the critical speed curve at the baseline configuration. These shape sensitivity derivatives give a linear approximation to the critical speed curve about the baseline configuration and will be useful for preliminary design or an optimization with aeroelastic constraints. The aeroelastic equations are also integrated with respect to time using the Wilson- θ method. One of the coefficients of the polynomial representing the transverse deflection is perturbed and by performing the time-integration the wing tip displacement is monitored as time progresses for different values of the freestream speed. The time integration of the aeroelastic equations shows the real-time aeroelastic response (tip deflection as a function of time) of the wing when operating at speeds at the verge of instability. Particularly notable is the response of the wing configuration where the divergence speed and the flutter speed are close to each other.

Flutter analysis of the wing is also carried out using a lifting-surface subsonic kernel function aerodynamic theory (FAST) and an equivalent plate structural model. The generalized aerodynamic forces are obtained for a fixed number of free vibration modes of the wing, for the specified Mach number and spanning the range of reduced frequencies. The flutter speed is obtained using a V-g type of solution. The novel method of automatic differentiation using ADIFOR was successfully implemented to generate exact derivatives of the flutter speed as obtained from the discrete system, with respect to shape parameters of the wing. A good sensitivity prediction is obtained using ADIFOR. Also, based on the sensitivity of flutter speed to modal parameters, namely, natural frequency, generalized mass and generalized aerodynamic forces, computed using ADIFOR, one could make a judicious choice of the number of modes to be used in the aeroelastic analysis for reasonably estimating the flutter speed. These derivatives give a qualitative estimate of the modes that actively participate in flutter. In order to obtain a quantitative estimate of the contribution of a particular mode to flutter, a parameter was formed by constructing a logarithmic derivative of the sensitivity to modal parameters and summing up the absolute values of these derivatives for each mode. The finitely vanishing value of the parameter for higher modes indicates that even if these modes are not used for the flutter calculation, the results would be affected only to an insignificant amount.

Finite element modeling of the wing is done using NASTRAN so that wing structures made of spars and ribs and top and bottom wing skins could be analyzed. The modeling is validated for static and dynamic analysis using a box-beam and an AGARD swept-back

wing model. A good agreement with previously published results is obtained. The airfoil shape for the wing cross-section was generated by transformation from a circle using the Joukowski transformation, so it could be parameterized. The free vibration modes of the wing obtained from NASTRAN are input into FAST to compute the flutter speed. The derivatives of flutter speed with respect to shape parameters are computed using a combination of central difference scheme and ADIFOR and the sensitivity to modal parameters are calculated using ADIFOR. A fairly good sensitivity prediction is obtained. It is seen that the higher vibration modes of the wing do not actively participate in flutter.

An equivalent plate model which incorporates first-order shear deformation theory is then examined so it can be used to model thick wings, where shear deformations are important. The sensitivity of natural frequencies to changes in shape parameters is obtained using ADIFOR which would be useful in optimization with frequency constraints. The natural frequencies and the flutter speeds calculated using the first-order shear deformation theory are compared with those obtained using a classical laminated plate theory. It is seen that the frequencies and the flutter speeds drop as the transverse shear effects come into play. The shape sensitivity derivatives of the flutter speed are indispensable in a gradient-based optimization with aeroelastic constraints. A simple optimization effort is made towards obtaining a minimum weight design of the wing, subject to flutter constraints, lift requirement constraints for level flight and side constraints on the planform parameters of the wing. The nonlinear constrained optimization problem is solved using the IMSL subroutine NCONG, which uses successive quadratic programming. An optimum design which satisfies all the constraints is obtained. It should be noted that more constraints can be added to this optimization problem as desired.

The quality of the sensitivity results of the aeroelastic response obtained by analytical derivatives and by automatic differentiation in this study indicates that these shape sensitivity derivatives could be useful in a preliminary or conceptual design, where several shape changes in the design are to be made, before the shape of the final configuration is frozen. The modal sensitivity derivatives of the flutter speed give a quantitative estimate of a reasonable number of modes that need to be used for a flutter analysis and enables one to make a judicious choice. In a gradient-based optimization with aeroelastic constraints or flutter constraints, the shape sensitivity derivatives are required and their usefulness is demonstrated by performing a simple optimization study.

Publications related to this project:

Thesis:

Issac, J.C., "Sensitivity Analysis of Wing Aeroelastic Responses," Ph.D Thesis, Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1995.

Eldred, L.B., "Sensitivity Analysis of the Static Aeroelastic Response of a Wing," Ph.D Thesis, Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1993.

Journal publications:

Issac, J.C., Kapania, R.K., and Barthelemy, J.-F.M., "Aeroelastic Sensitivity Analysis of Wings Using Automatic Differentiation," Submitted to *AIAA Journal*. Also submitted to the 6th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Sept. 4-6, 1996, Bellevue, WA.

Eldred, L.B., Kapania, R.K., and Barthelemy, J.-F.M., "Sensitivity Analysis of Aeroelastic Response of a Wing Using Piecewise Pressure Representation," To appear in *Journal of Aircraft*.

Issac, J.C., Kapania, R.K., and Barthelemy, J.-F.M., "Sensitivity of Flutter Response of a Wing to Shape and Modal Parameters," *AIAA Journal*, Vol. 33, No. 10, Oct. 1995, pp. 1983-1986.

Bhardwaj, M., and Kapania, R.K., "Shape Sensitivity Analysis of Divergence Dynamic Pressure," *Journal of Aircraft*, Vol. 32, No. 4, 1995, pp. 898-901.

Kapania, R.K., and Issac, J.C., "Sensitivity Analysis of Aeroelastic Response of a Wing in Transonic Flow," *AIAA Journal*, Vol. 32, No. 2, Feb. 1994, pp. 350-356.

Kapania, R.K., Eldred, L.B., and Barthelemy, J.-F.M., "Sensitivity Analysis of a Wing Aeroelastic Response," *Journal of Aircraft*, Vol. 30, No. 4, 1993, pp. 496-504.

Conference proceedings:

Issac, J.C., and Kapania, R.K., "Time-Domain Aeroelastic Response of an Aircraft Wing in Compressible Flow," Proceedings of the International Conference on Structural Dynamics, Vibration, Noise and Control, Hong Kong, Dec. 5-7, 1995, pp. 442-447.

Issac, J.C., Kapania, R.K., and Barthelemy, J.-F.M., "Sensitivity Analysis of Flutter Response of a Wing Incorporating Finite-Span Corrections," Presented at the 5th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Panama City, FL, Sept. 1994.

Kapania, R.K., Issac, J.C., and Barthelemy, J.-F.M., "Sensitivity Analysis of Flutter Response of a Typical Section and a Wing in Transonic Flow," Presented at the 34th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, La Jolla, CA, April 19-22, 1993.

Eldred, L.B., Kapania, R.K., and Barthelemy, J.-F.M., "Sensitivity Analysis of Aeroelastic Response of a Wing Using Piece-wise Pressure Representation," Presented at the 34th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, La Jolla, CA, April 19-22, 1993.

Kapania, R.K., "Sensitivity Analysis of Dynamic Aeroelastic Responses," *AGARD Workshop on Aeroelasticity*, Bath, England, May 1-3, 1991, in Integrated Design Analysis and Optimization of Aircraft Structures, AGARD Report 784, pp. 3-1 to 3-12.

Kapania, R.K., Eldred, L.B., and Barthelemy, J.-F.M., "Shape Sensitivity Analysis of Static Aeroelastic Response," Presented at the 32nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Baltimore, MD, April 1991.

Kapania, R.K., and Barthelemy, J.-F.M., "Sensitivity Analysis of a Wing Aeroelastic Response," ICASE-17, Stockholm, Sweden, Sept. 1990.